Patterns, Functions and Equations: Cognitive, Concrete Connections

Brief Overview:

This unit focuses on connecting patterns to functions and equations. The students will complete a series of patterns based on their own names in order to create function tables that express these patterns numerically. The students will extend their understanding of function tables using common real-world objects, analyzing their relationships graphically and numerically. Finally, students will generate full equations with variables.

NCTM Content Standard/National Science Education Standard:

<u>Algebra</u>

Understand patterns, relations, and functions

- **Represent and analyze patterns and functions, using words, tables, and graphs.**

  Represent and analyze mathematical situations and structures using algebraic symbols
- **Represent the idea of a variable as an unknown quantity using a letter or a symbol.**
- **Express mathematical relationships using equations.**

  Use mathematical models to represent and understand quantitative relationships
- **Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.**

Grade/Level:

This unit is appropriate for fifth grade.

Duration/Length:

This unit requires three 60-minute class periods.

Student Outcomes:

Students will:

- **Represent patterns in numeric form**
- **Create word and number expressions and equations**
- **Express mathematical relationships using equations**
- **Understand that quantities on both sides of an equation must be equal**
- **Use the following vocabulary correctly: pattern, rule, function table, variable, expression and equation.**

Materials and Resources:

Lesson 1

- **A class set of the 5 x 5 square grid (Student Resource 1)**
- **Approximately 10 each of: 8 x 8 square grid, 7 x 7 square grid, 6 x 6 square grid, 4 x 4 square grid, and 3 x 3 square grid (Student Resources 2, 3, 4, 5, and 6)**
- **Crayons or markers**
- **Tape**
- **Teacher Resource 1**
- **Graph paper (for Extension activity only)**

Lesson 2

- **Chart Paper**
- **Markers**
- **Scissors**
- **Glue**
- **Geometric Shapes (for Reteaching only)**
- **Tape**
- **Student Resource 7 for each cooperative group**
- **Student Resources 8A and 8B (photocopy front/back) for each pair of students**
- **Teacher Resources: 2B, 3A, 3B, 4A, 4B, 5A, 5B, 6A and 6B (one each) and 2A (two copies).**
  - **These should be pre-cut and sorted to save time. See directions in the lesson**

Lesson 3

- **Scale Mat (Student Resource 10)**
- **For each pair of students, a set of geometric shapes (at least 5 each of 4 shapes)**
- **Balance Worksheet (Student Resource 11)**
- **For each pair of students, a set of counters (at least 10 each of 3 types)**

Development/Procedures:

Lesson 1
Pre-Assessment

- **Play a game of "Hot Potato" where students attempt to pass an object around the room one time within 10 seconds. (The intent is for students to "fail" so changes must be made.) When students fail, *ask: What changes can you make?* (lengthening the time, having students move more closely together, removing students, etc.) Attempt the game a second time with the new criteria until students are successful.**

(The goal is to encourage students to make adjustments to achieve a desired outcome.)

Launch

- **Distribute the 5 x 5 square grids on Student Resource 1 and direct students to write their first name on the grid, one letter per box. Students should continue writing their name even if it wraps around a line until the grid is full.**
- **Next, instruct students to color in every square that contains just the first letter of their name. (See example on Teacher Resource 1).**
- **When the first student finishes, ask him/her to tape the grid onto the board. When the second student finishes, have him/her tape the grid under the first grid if the pattern matches, or start a new column for their pattern. As students finish, they should tape their grids under patterns that match their own or start new columns for new patterns.**

Teacher Facilitation

- **Direct students to share in their groups the similarities and differences among the patterns.**
- ***Ask: How are the patterns in (this) group alike? Is (this) group different from (that) group? How? Why is this true if everyone had the same size grid?*** **(Answers will vary.)**
- **Point to the sets of patterns that create a vertical stripe (Pattern A in Teacher Resource 1) and *ask: What is this pattern? Is there a relationship between the names that all made these patterns?***
- **Teacher should direct the discussion that a vertical stripe is created when the number of letters in a name is the same as the number of squares in a row.**
- ***Ask: How many squares would be needed to make the vertical stripe using your own name?***
- **Teacher should now start a vertical function table with columns labeled "# of Letters in Name" and "# of Squares per Row." The first entry set should be (5,5).**
- ***Ask: If your name has 4 letters in it, how many squares does your grid need to create the same pattern?*** **(You need 4.)**
- **Continue this table with all the number relationship scenarios the students suggest.**

Student Application

- **Ask students to choose their favorite pattern from the options on the board and predict which size grid they need to match their name to**

that pattern. Students may not choose Pattern A or the pattern they created first. Allow students time to share their predictions with students in their group.

- **Distribute students' grid choices and have them repeat the process of filling in the grid with their name and coloring in the first letter of their name. See Student Resources 2-6.**
- **When students complete their grids, hang them on the board in the column that matches the pattern they created. Many student predictions may be wrong so students should hang grids with the pattern that matches not with the pattern they intended.**
- ***Ask: How are the grids in (this) group alike? Is (this) group different from (that) group? How? Why is this true if everyone had the same pattern?***
- **Lead students to discover that the pattern is determined by the relationship of letters in the name and of squares in the grid.**
- **Teacher should now start a vertical function table with columns labeled "# of Letters in Name" and "# of Squares per Row" for each of the patterns. Choose one pattern and model how to complete the table for that pattern including all of the entry sets.**
  - **See example pattern charts and matching function tables (Teacher Resource 1)**
- **Students should complete similar function tables for the remaining patterns. Observe student tables and choose volunteers to draw their tables under the appropriate patterns**
- ***Ask: If you like Pattern (X) the best and your name has (4) letters in it, how many squares in your grid row would you need?* Continue to ask similar questions with varying patterns and letter combinations. *Ask: Is there a relationship between the number of letters in a name and the number of squares in a row that generates a specific pattern?***
- **Lead students to verbalize a rule that matches each function table. (ex.. To make Pattern X, I need to have one more square than the number of letters in my name.) It does not need to be a formal equation (*n* +1).**

Embedded Assessment

- **Observe students as they predict which grid is needed to create their preferred pattern.**
- **Observe students as they describe rules/patterns for each pattern type.**

Reteaching/Extension

- **Struggling students can be redirected in a small group. Use the same three-letter word to complete, in order, a 3 x 3, 4 x 4 and 5 x 5 grid. After each grid, have students verbalize the pattern they see, how many letters and squares are in a row and how this differs from other grids. Work with students to complete a function table for each grid.**
- **For students who understood the lesson, give them graph paper and instruct them to choose their favorite pattern and apply a new scenario to it (last name, acronym, pet's name, etc.). When finished, have them write in words how they created that pattern and the relationship between the number of letters and the number of squares.**

***Name pattern activity in this lesson was adapted from *A-Plus Math Solutions* by Marylin Burns.**

Lesson 2

Pre-Assessment

- **Create a vertical function table on chart paper or on the board that contains the number sets: (3,6), (5,10), and (4,8). Write the number sets in this order so students do not draw conclusions about the relationship between the group of input values and the group of output values, instead focusing on the relationship between the input and output columns. Cover the chart so only the heading and first number set is revealed (3,6). Then, make four large signs with the following: "x2" "÷2" "+3" and "-3." Hang one "answer station" on each wall of the classroom so students can stand under them.**
- ***Ask: Do you see a relationship between these numbers (3, 6)?* Students will give a variety of answers; do not tell them if they are correct, though note their answers as part of an assessment of their understanding.**
- **Each student should stand under the answer station they think contains the correct relationship.**
- **Once students are under the signs, *ask: Why did you choose _____?* Be sure to review why "÷2" and "-3' are incorrect and allow students under those signs the opportunity to move. After several students explain their reasoning, reveal the next set of numbers in the table (5,10).**
- ***Ask: Think again about the expression you chose. Does it work for these new numbers?* Allow students think time and then instruct them to move to a new answer station if they have changed their mind.**
- **For each group, *ask: Why did you choose _____?* Review why "+3" does not work anymore. Allow students to move as their thinking changes. After several students explain their reasoning, reveal the next set of numbers in the table (4,8).**

- ***Ask: Does this set of numbers continue to follow the pattern we identified of "x2"?***

- **Divide students into 4 or 5 cooperative groups. Distribute to each group: a copy of the Input/Output chart (Student Resource 07), a large input/output function table on chart paper (if chart paper is not available you may use an additional copy of Student Resource 07), a set of picture pairs (faces & eyes, faces & hands, dogs & paw prints, roosters & feet, houses and windows) and glue.**
  - **Teacher should print, laminate (if desired), and pre-cut matching sets of pictures (Teacher Resources 02B, 03A, 03B, 04A, 04B, 05A, 05B, 06A and 06B. Example: 1 dog 1 set of 4 paws, 2 dogs & 2 sets of 4 paws.**
- ***Ask: What do you know about the relationship between people and ears?* (People usually have two ears.)**
- ***Ask: How many ears do (Student A) and (Student B) have together?* Say, "This pattern sounds like it will make a rule, so I'm going to write it down on a table." Draw a table on the board that tracks the relationship between people and ears. Explain that this table is called a function table and is a mathematical way of displaying the numerical relationship between two things. Explain that function tables are typically vertical and contain the headings "INPUT" and "OUTPUT." Have students direct the teacher to write the number of people in the input column, and the number of ears in the output column.**
- **Direct the students to use the pattern pairs you distributed to create a function table. Each picture has a relationship with the other picture in its set, so students should discuss the relationship within their group.**
  - **Note: Students may choose eyes as input and faces as output or vice versa. Both are correct and the key component is that the students remain consistent placing the information in the columns. Students may also arrange the data in any order; it does not need to be in a hierarchy (ex., 1 face then 2 faces then 3 faces, etc.)**
- **Instruct students to glue the pictures onto the function table so the relationship follows the pattern or rule they observed.**
- **Instruct students to then fill in their function tables correspondingly (This will be the second function table for teachers not using chart paper). That is, if the students glue one pair of eyes under input and one face under output on the chart paper, they should write 2 under input and 1 under output on the function table.**

Teacher Facilitation –

- **Display the completed picture pair function tables and matching function tables. Choose one group and function table to begin with. Ask the group to explain their work.**
- ***Ask: What is the relationship between the number of ___ and the number of ___ ? Is this true for each of your entries? How many _____ will you get in the output if you have 10 ____?***
- **Explain that these relationships are called "rules" and every function table has one. Ask this first group to write a "rule" for their picture pairs. Direct students to use math vocabulary (add 2, multiply by 5, subtract 3, etc.)**
- **Model how to write that rule as an equation. *Ask: If I'm not here, you have a _____ teacher?* (Substitute) Say, "Mathematicians need to substitute things in math sometimes too. If we don't know a number, we use a substitute that we call a variable. *Ask: Have you heard this word before? Tell me what you know about this word.* (A variable is a letter that stands for a number. I have heard it in math and science.)**
- **Allow for a short discussion of the term variable and focus on the mathematical use today. After explaining that variables are often letters, suggest that "d" be used for the number of dogs. Write: "Rule: d x 4" on the Student Resource ?. Allow time for questions.**
- **Student Resource 7. Allow time for questions. Continue to analyze the charts and add rules under student direction with teacher support as necessary until all charts are correct and complete.**
- ***Ask: How do I know how many input (dogs/paws) I will have if I know the output (dogs/paws)?* The object is to reverse the order of thinking so that students see that the relationship works both ways.**
- ***Ask: What if I don't know how many (dogs) I want? What can you tell me about the number of (paws) I will have? What will it always be?* Students should be able to respond that there will always be 4 paws for each dog. *Ask: How can you write this?* (*p = d* x 4)**
- ***Ask: Think about each chart. Can you explain if this is true for every chart or only some?* Allow for student responses. If students seem confused, cover up the headings and flip the chart upside down so that they can observe the relationship will still exist. Discuss observations.**

Student Application

- **Divide students into pairs for this activity.**
- **Distribute Student Resource 8A and 8B (copied front/back) and scissors. Instruct students to cut along the dotted vertical lines and then fold along the solid, horizontal line.**
- **Direct the student pairs to generate four function tables on the front worksheet. The student pairs should think of the rule, but only write**

number pairs in the table that will fit the rule. Students must use a minimum of three full number sets.

- **Instruct students to leave one number missing from a set so that someone else will have to "find the rule" of their table based on their number sets.**
- **Once students have completed filling in all four function tables with four different rules each student should write his or her name on the inside of the template in the "Created By" space.**
- **As students finish, have pairs swap templates and solve the function tables by writing the missing number in the answer space and the rule (ex: y-5) in the rule space. The student pair that solves the tables should write their names in the "Solved By" space.**

Embedded Assessment

- **Observe students as they choose answer stations during the pre-assessment, particularly during the final number set.**
- **Observe and record student performance during cooperative function table activity.**
- **Monitor and engage students during the Student Application portion of the process or collect papers for evaluation.**

Reteaching/Extension

- **Struggling students can be redirected in a small group. Give each student Student Resource 07, and a manipulative triangle. Discuss how many sides the shape has. Have students draw/trace the triangle on the input side of the template and write 3 in the output side. Give the students another triangle and repeat the steps, filling in the function table.**
- **See Student Resource 9. (Answer key: 1. 10 2. 16 3. 15)**

Lesson 3

Pre-Assessment

- **Tell students they are going to play a game of tug-of-war. Begin to assign students in the class to teams. As you call the teams, have the students stand together. When you choose students, be sure to create an unfair situation (ex., most of the students v. few students; all of the larger students v. smaller students).**
- **Once students see the teams, *ask: Are you ready to play?* Facilitate a discussion about why some students don't want to play since the teams are unfair. *Ask: What is unfair about this scenario?* (Possible answer:**

too many people, all the strong people, etc.) *Why is this unfair?* (The teams need to be even or equal.) *What can we change?* (Each team needs the same number of people.)

Launch

- **Explain to students that the focus of the previous day's lesson was to identify and describe relationships between picture scenarios and then track the patterns and rules in function tables.**
- **Write the following on the board: $y + 6$, $8 = y + 6$**
- ***Ask: What is different about these two entries?* (One has an 8 and an equal sign and one doesn't).**
- **Explain that $y + 6$ is called an expression or a rule but that $8 = y + 6$ is called an equation because it has an equal sign.**
- **Group students in pairs and distribute a Scale Mat (Student Resource 10) and a set of geometric shapes**
  - **Sets should include at least 2 or 3 each of multiple shapes (ex., triangles, squares, rectangles, hexagons, etc.)**
- **Draw a see-saw on the board and model one scenario on the board. For example, draw one hexagon on one side of the scale. *Ask: What do you know about a hexagon?* (It has 6 sides.) *If I wanted to balance the scale so that the numbers of sides are equal, what shapes could I place on the other side of the scale?* (2 triangles) Have students complete this as you model it on the board.**
- ***Ask: Which math term means the same as balance?* (Equals)**
- **Give students the following scenarios and have them complete the matching shapes:**
  - **3 triangles (3 triangles OR 1 hexagon and 1 triangle)**
  - **2 squares (1 octagon OR 2 rectangles OR 1 triangle and 1 pentagon, etc.)**
  - **3 squares (1 octagon and a square OR 4 triangles, etc.)**
- **As the class works through each scenario, have volunteers share their answers. Continue with additional scenarios until students have grasped the idea of creating equal values on each side of the scale.**

Teacher Facilitation

- ***Ask: What is a variable?* (a letter that stands for a number)**
- ***Ask: What variable should we assign to the shapes you have in front of you?* (H for hexagon, T for triangle, S for square, etc.)**
- **Model again the first scenario (1 hexagon and 2 triangles) on the board. Under the pictures, write the numeric expression that matches ($h = t + t$). Call on a student to verbalize the expression (h equals t**

plus t). Explain to students that another expression could also be *h* = 2*t*

- **Once you have *h* = *t* + *t* on the board, *ask: What does h equal?* (6) Replace H with 6 (6 = *t* + *t*). *Ask: What must t equal?* (Students should say 3 because they already know that *t* is a triangle). Write out the new equation: 6 = 3 + 3. Remind students that since T is the same letter, the same number must represent it.**
- **Repeat this activity with each of the scenarios the students created in the launch activity.**
  - **Example: 3 triangles equals 1 hexagon and 1 triangle**
  - **First, write the expression with just variables (*t* + *t* + *t* = *h* + *t)***
  - **Next, assign a value to each variable (*t* = 3, *h* = 6)**
  - **Then, write the numeric equation (3 + 3 + 3 = 6 +3**
  - **Finally, solve the equation (9 = 9)**
- **Have student volunteers come to the board to write out the steps under the appropriate scenario.**

Student Application

- **Distribute to students the Balance Worksheet (Student Resource 11) and three types of counters (at least 10 of each)**
- **Model for students how to complete the first problem (see the Student Resource) and then have them solve the rest of the problems independently or in pairs.**
- **When students complete, take time to review each of the answers.**
- **Important points to make:**
  - **Sometimes, there can be more than one answer to a problem**
  - **All equations must balance**
  - **Different variables must equal a different variable**

Embedded Assessment

- **Observe student responses to the tug-of-war game to determine that they understand fairness and equality**
- **Observe students as they match up the geometric figures on the scale**
- **Student responses on the balance worksheet can be collected for evaluation.**

Reteaching/Extension

- **Struggling students can be pulled into a small group and use additional types of manipulative to reteach lesson**

- **For students who understand the concept, create scenarios with money that students have to balance. For example, $0.25 = $0.10 + $0.10 + $0.05. Encourage them to write out an expression with variables and with numbers for each scenario.**

Summative Assessment:

- **Student Resource 12**
- **Teacher Resource 7 (Answer Key)**
- **This assessment will evaluate the students' understanding of function tables and writing equations with variables and numbers**

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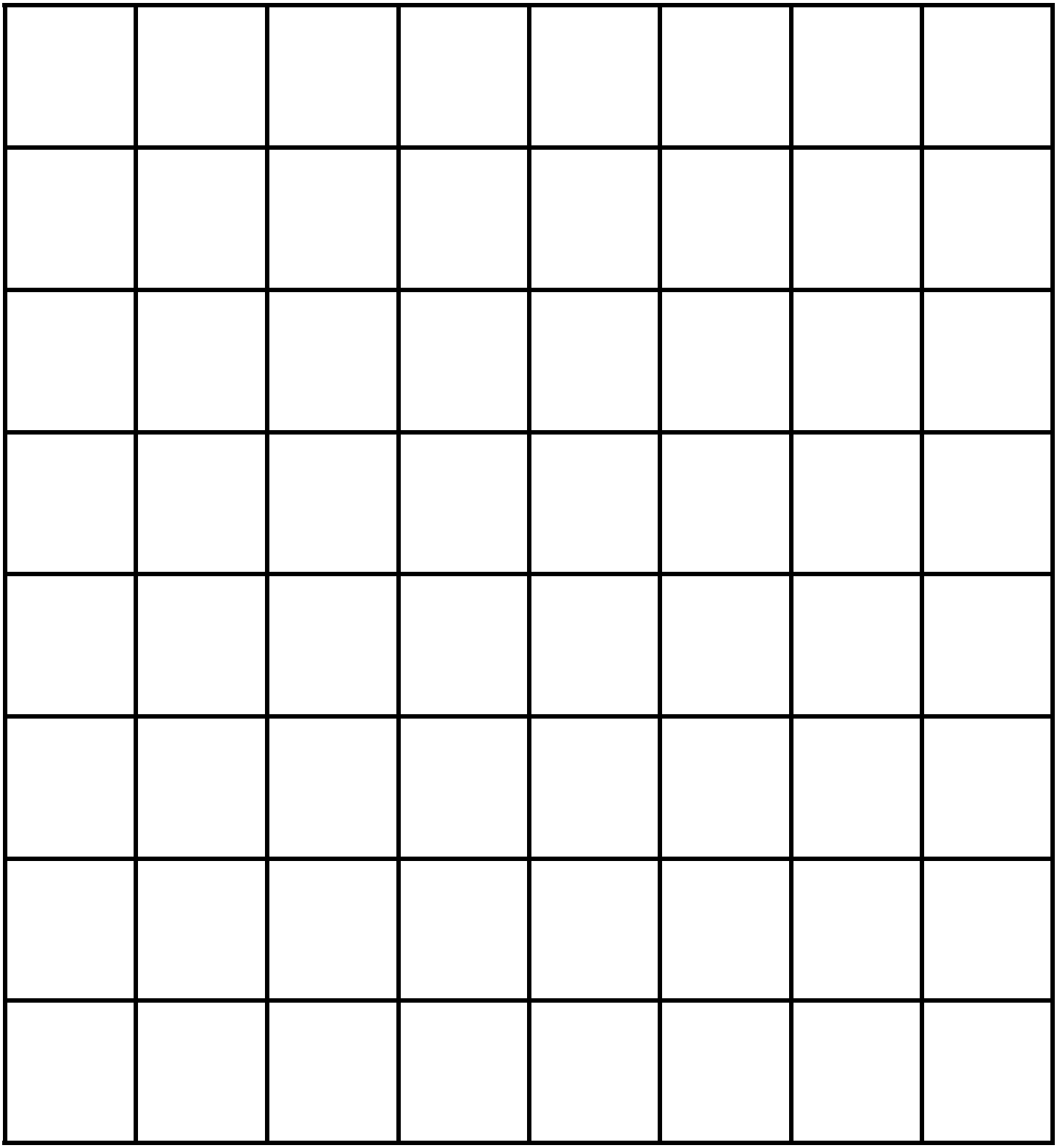
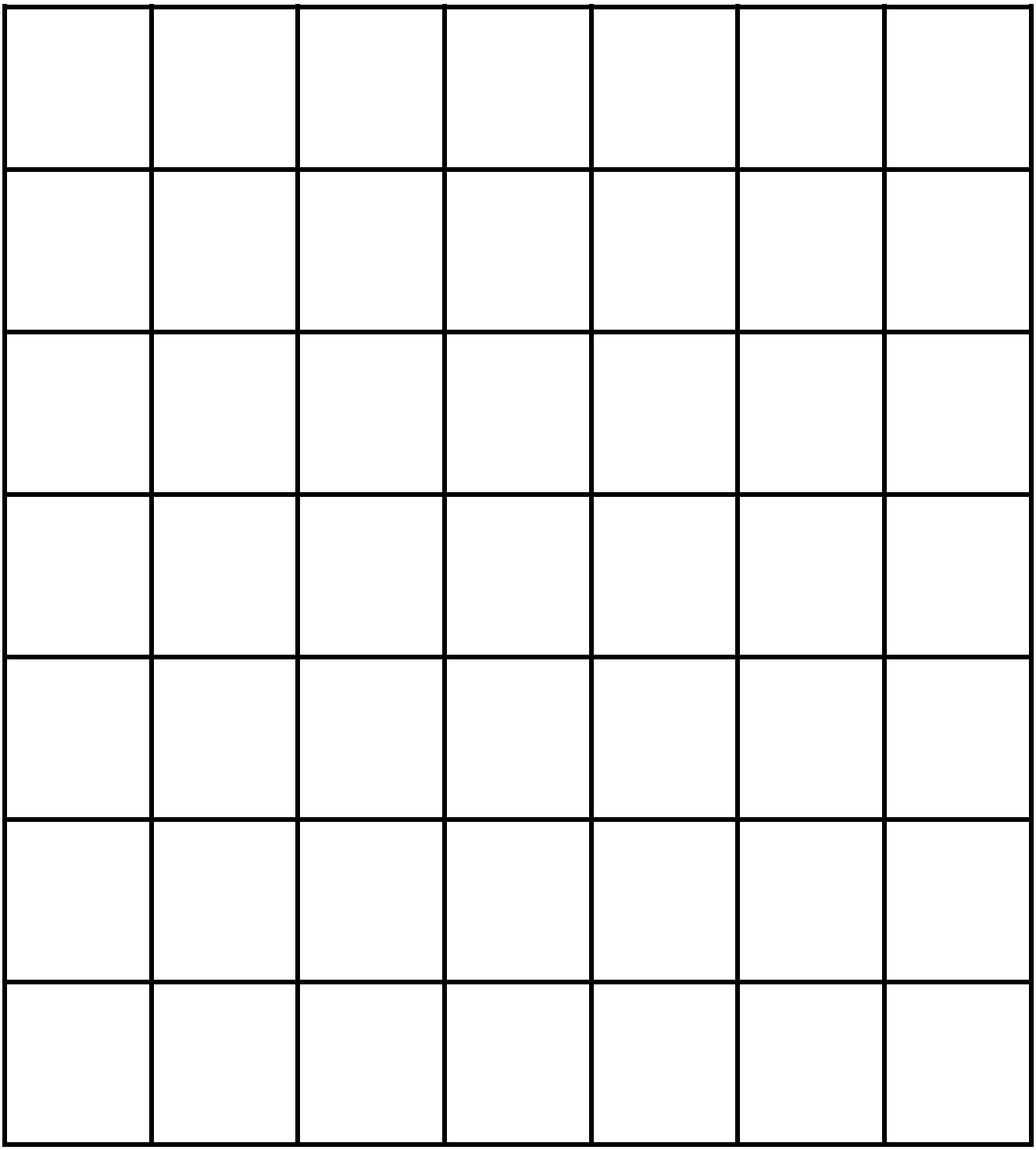
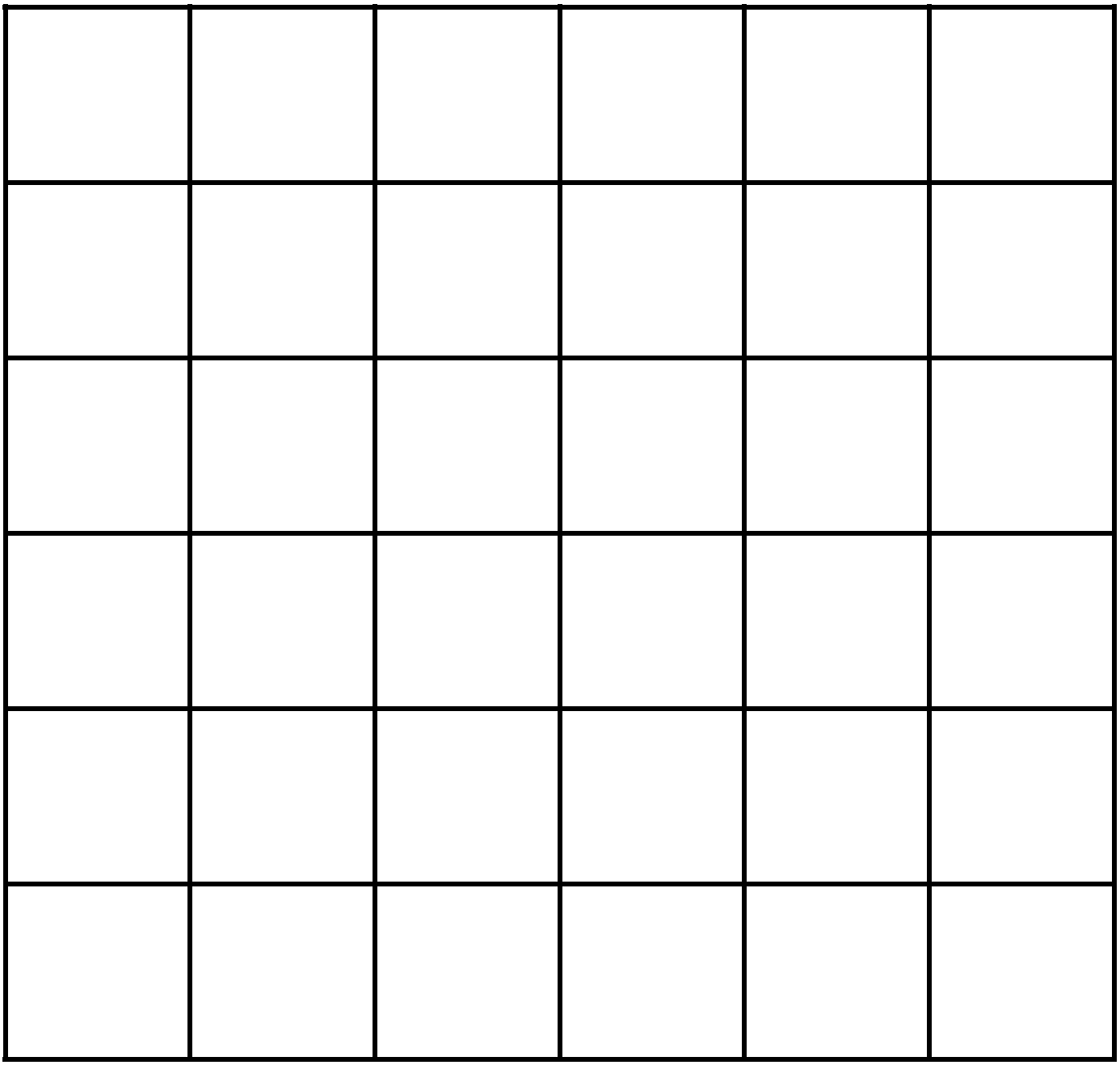






INPUT	OUTPUT

INPUT	OUPUT_

INPUT	OUPUT_

INPUT	OUPUT_

INPUT	OUPUT_

Created By _____ _____ Solved By _____	Created By _____ _____ Solved By _____	Created By _____ _____ Solved By _____	Created By _____ _____ Solved By _____
Answer _____ Rule _____	Answer _____ Rule _____	Answer _____ Rule _____	Answer _____ Rule _____

Name _____ Date _____

Function Tables – What's the Rule?

1. **The elephants are buying tickets for "Mystery of the Secret Trunk." Selma pays for three tickets and gets seven tickets! Gus pays for one ticket and gets five tickets! Gina pays for four tickets and gets eight tickets! What in the world is going on? The elephants then watch Ken pay for six tickets. How many tickets will Ken get?**

2. **Strange things are happening in the Hamster House Hotel. Dawn gets on the elevator and presses button 10 but the elevator goes to floor 5! Jose presses button 8 but the elevator goes to floor 3! Maria presses button 11 but the elevator goes to floor 6! What is going on at the hotel? If Dawn were to get back on the elevator and press button 21, where will the elevator take her?**

3. **Tyiesha is having a terrible time at Donut Delight. She has a machine that shoots out fresh donuts for each order entered at the register, however, today something is definitely wrong. Tyiesha punched in four donuts and 11 came out of the machine! Next, she ordered eight donuts but 19 donuts came out! Then, she entered seven donuts and 17 came out of the machine! If Tyiesha ordered six donuts, how many will the machine make? (Hint: There are two steps to this rule.)**

***Adapted from *Beginning Algebra Thinking For Grades 3-4* by Shirley Hoogeboom & Judy Goodnow and *Beginning Algebra Thinking For Grades 5-6 by* Judy Goodnow**

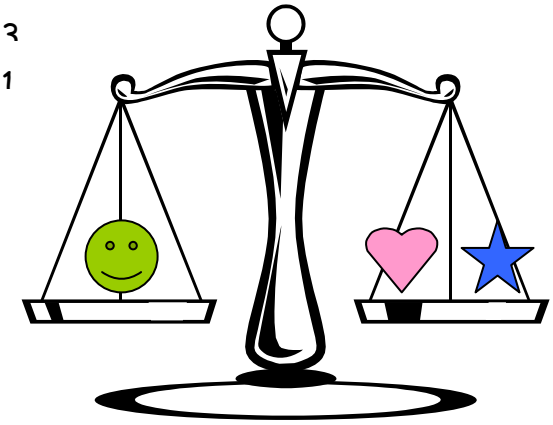
Name _____ Date _____

What Makes It Balance?

Use each scale as a guide to answer the set of questions. The first one has been done for you.

1. If ☺ equals 4 pounds, what could the other symbols weigh?

☺	♡	☆
4	1	3
4	3	1

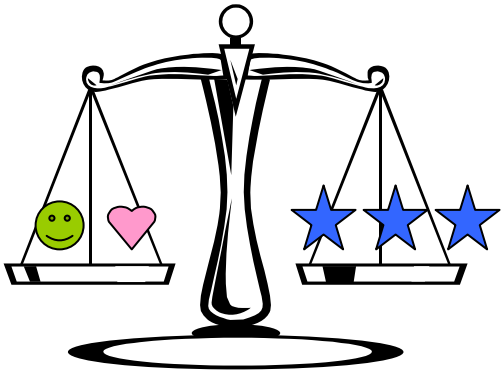


2. If ☺ equals 7 pounds, what could the other symbols weigh?

☺	♡	☆
7		
7		

3. If ☆ weighs 3 pounds, what could the other symbols weigh?

☺	♡	☆



4. If ☆ weighs 4 pounds, what could the other symbols weigh?

☺	♡	☆

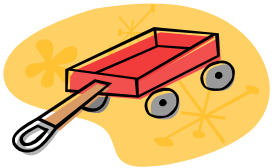
Name _____ Date _____

The Heavy Hogs Motorcycle Club is planning a day-long ride to raise money for local schools. Participants can ride in or on any vehicle, and each tire on the vehicle earns $0.10. Create a function table below to show the relationship between the vehicle and the total value of its tires.

Unicycle

Bicycle

Tricycle



Wagon

Steven's family rode different vehicles than Devyn's but earned the same amount of money as Devyn's family during the ride. Devyn's family earned an amount equal to T + B. If the variables T= tricycle, B = bicycle, U = unicycle, and W = wagon, write an equation using variables to show what vehicles Steven's family could have used.

Answer _____

Use what you know about variables and equations to explain how you know your answer is correct. You may use words, pictures, and/or symbols in your answer.

PATTERN A

F	I	R	S	T		N	A	M	E		S	E	T
F	I	R	S	T		N	A	M	E		S	E	T
F	I	R	S	T		N	A	M	E		S	E	T
F	I	R	S	T		N	A	M	E				
F	I	R	S	T									

# of Letters in Name	# of Squares per Row
5	**5**
4	**4**
3	**3**
n	**n**

PATTERN B

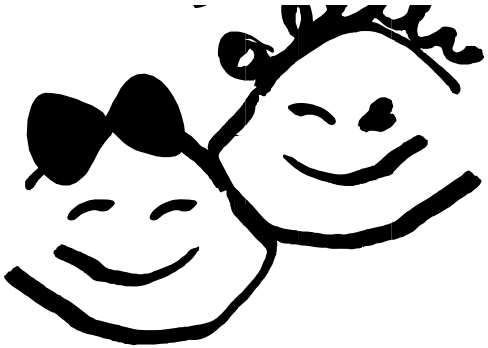
S	E	T	S		N	A	M	E	N		F	I	R	S	T	F
E	T	S	E		A	M	E	N	A		I	R	S	T	F	I
T	S	E	T		M	E	N	A	M		R	S	T	F	I	R
S	E	T	S		E	N	A	M	E		S	T	F	I	R	S
					N	A	M	E	N		T	F	I	R	S	T
											F	I	R	S	T	F

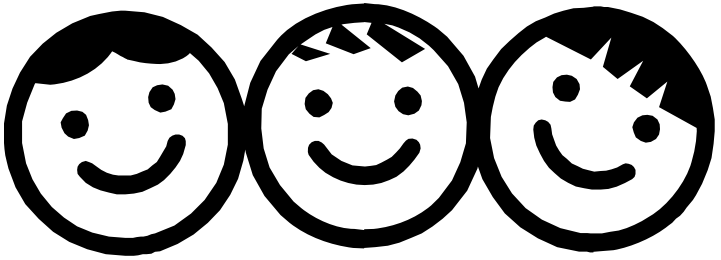
# of Letters in Name	# of Squares per Row
5	**6**
4	**5**
3	**4**
n	**n + 1**

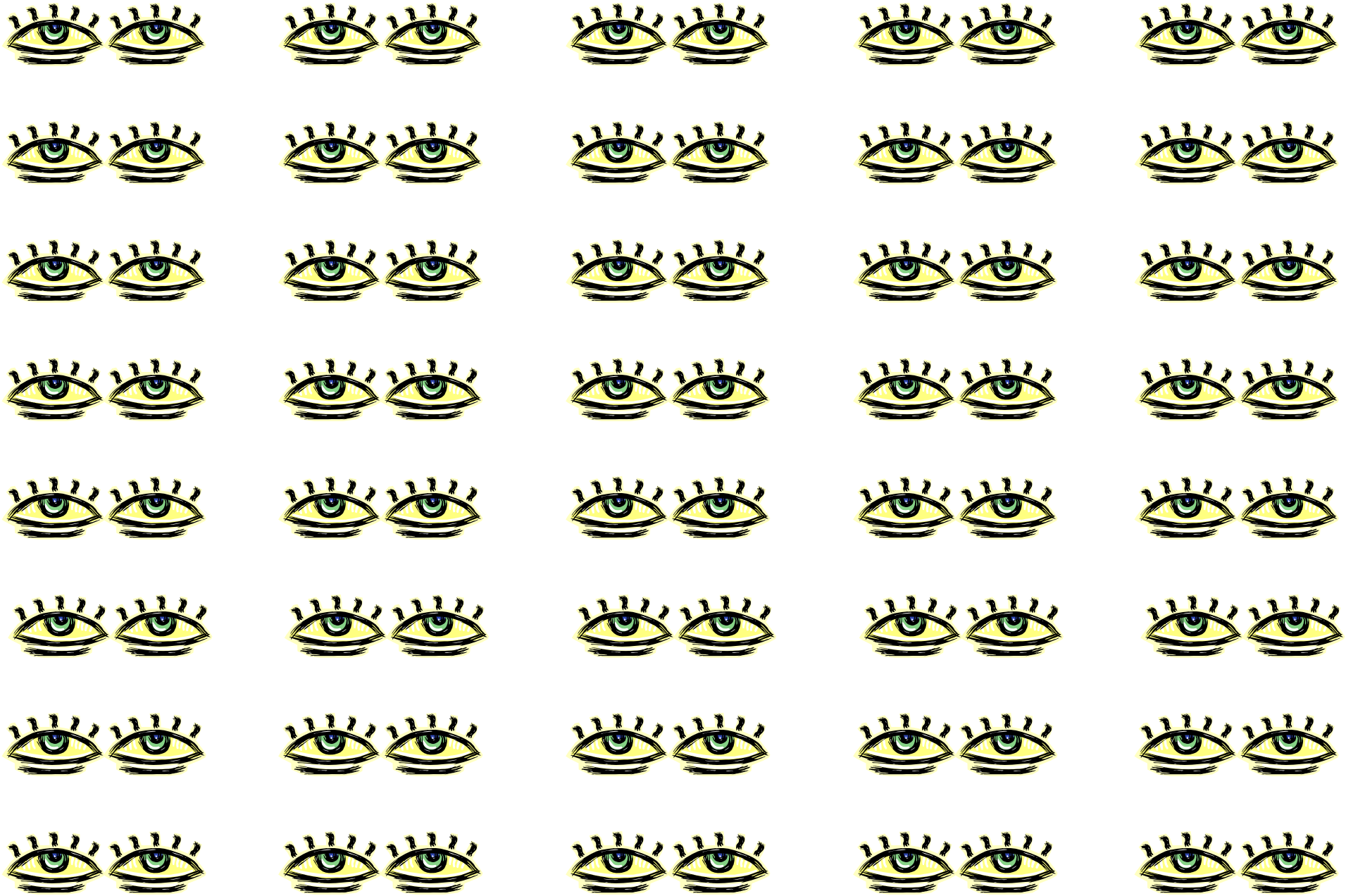
PATTERN C

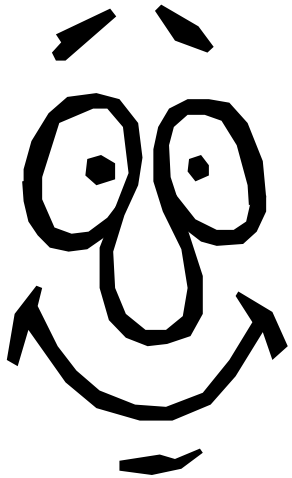
S	E		N	A	M		F	I	R	S	
T	S		E	N	A		T	F	I	R	
			M	E	N		S	T	F	I	
							R	S	T	F	

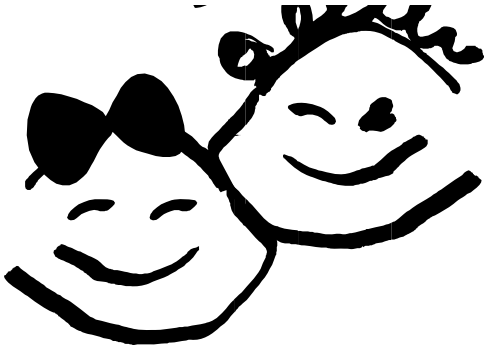
# of Letters in Name	# of Squares per Row
5	**4**
4	**3**
3	**2**
n	**n - 1**

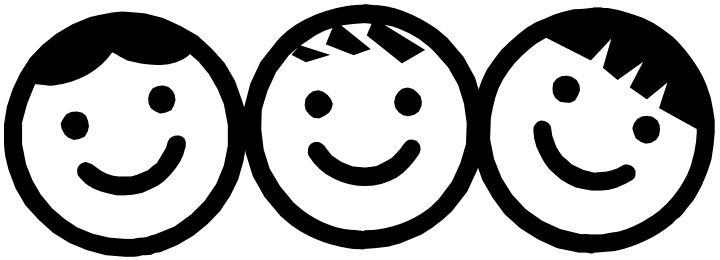


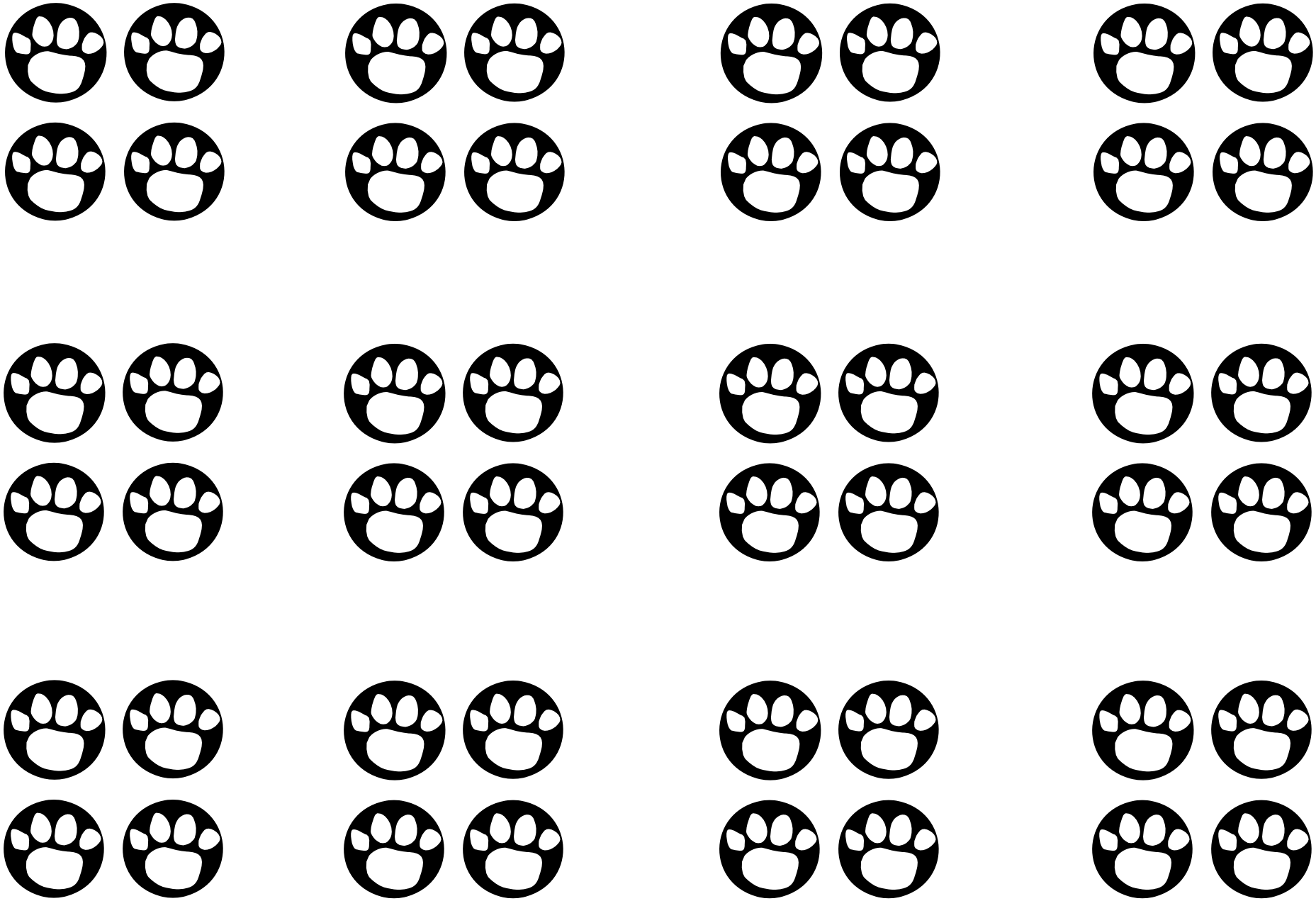


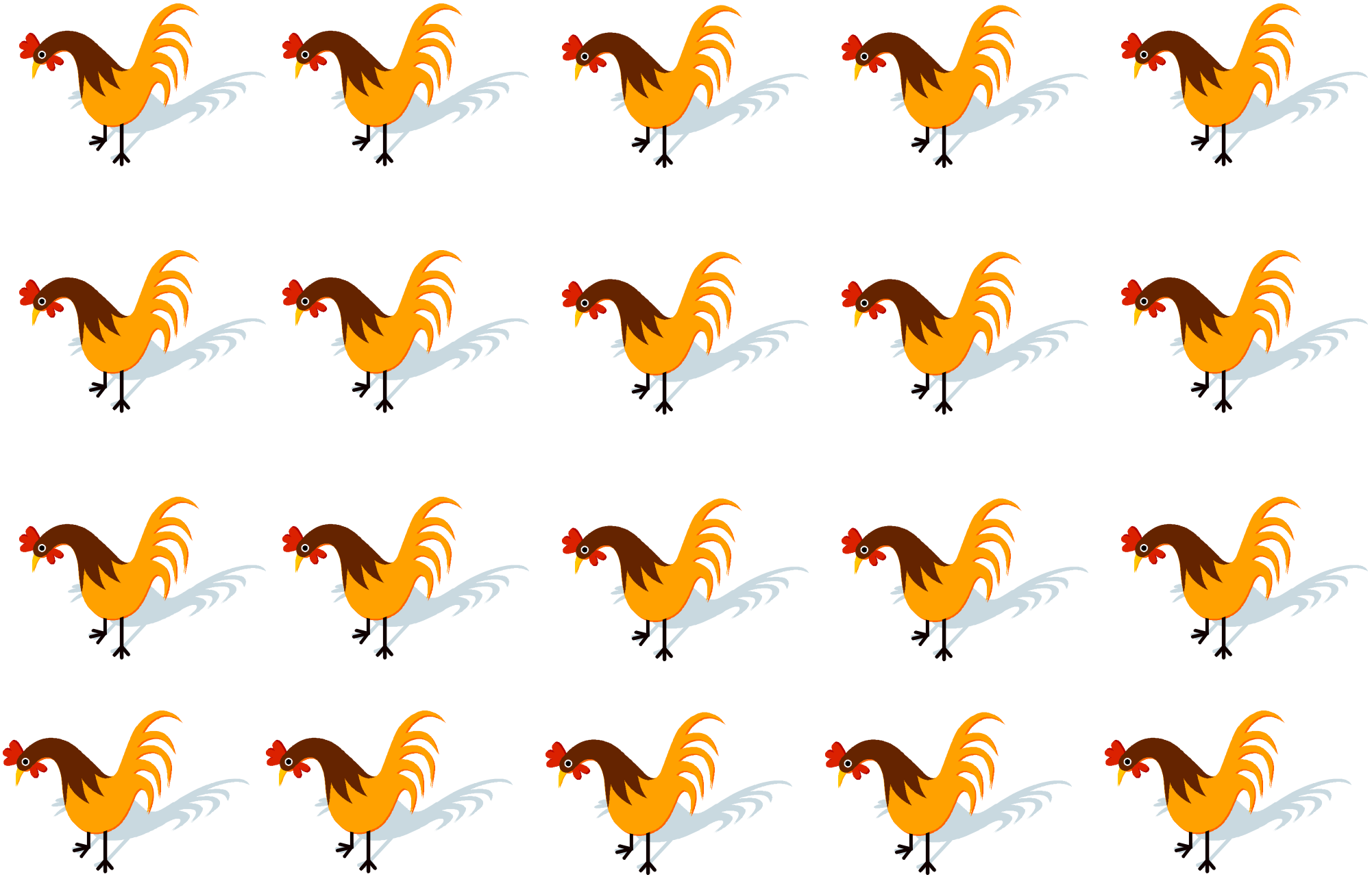


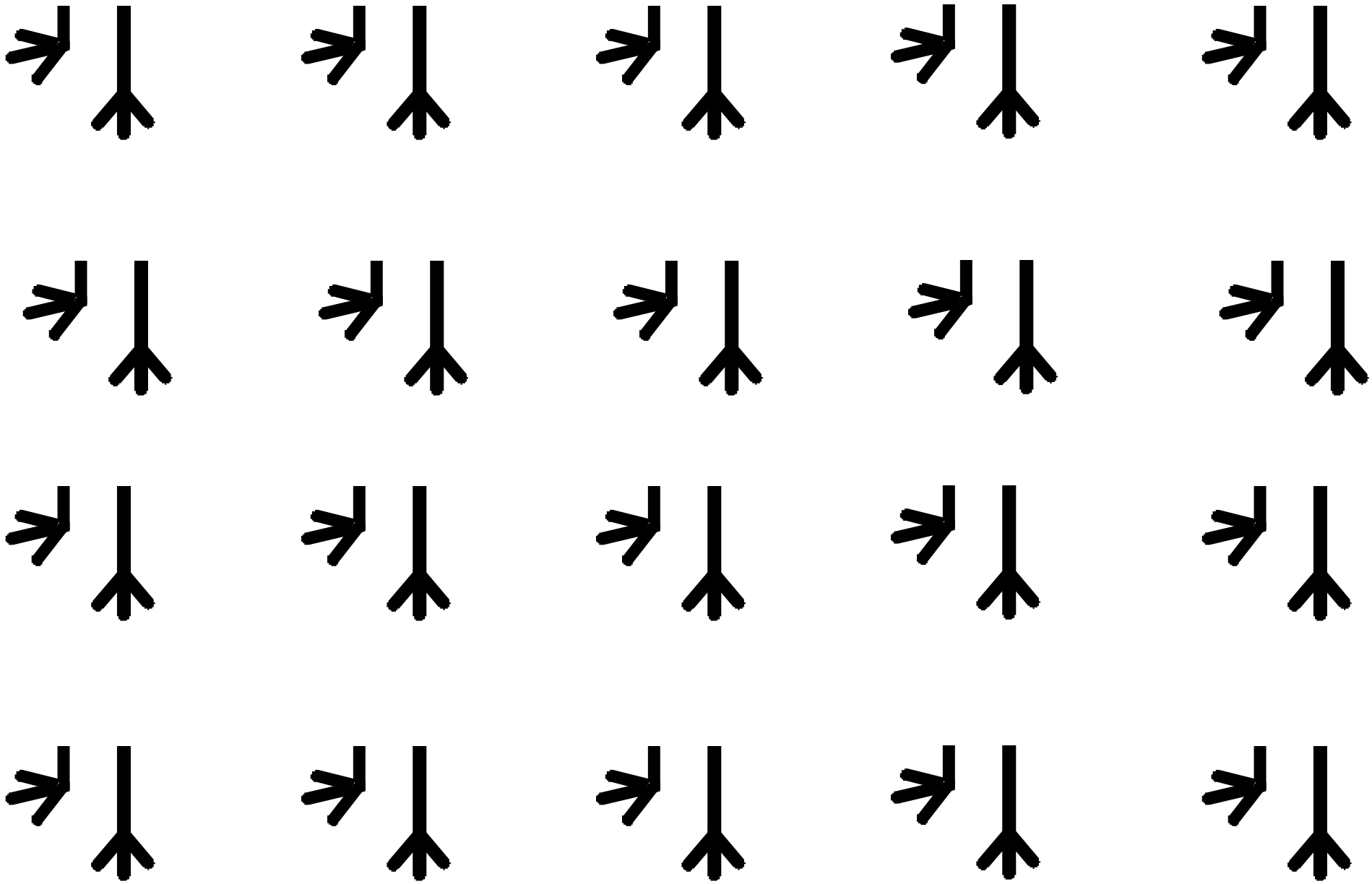












Summative Assessment Answer Key

1.

Input/ Vehicle	Output/ # of Tires
Unicycle	**1**
Bicycle	**2**
Tricycle	**3**
Wagon	**4**

Input/ # of Tires on Vehicle	Output/ Value of Tires
Unicycle 1	**$0.10**
Bicycle 2	**$0.20**
Tricycle 3	**$0.30**
Wagon 4	**$0.40**

2a. **Possible Answers:**

B + T = **U+U+U+U+U**
B+B+U
B+U+U+U
T+U+U
W+U

Students must **include the B+T=. Multiplication is also acceptable (5U; B+3U).**

2b. **Possible solutions should include the substitution of numeric values for the variables.**

T=.30
B=.20
U=.10

T+B=.30+.20
B+B+U=.20+.20+.10

T+B=B+B+U
.30+.20=.20+.20+.10
.50=.50

Each tire is valued at $0.10. A tricycle has 3 tires so its value is $0.30, a bicycle with 2 tires has a value of $0.20, and a unicycle with 1 tire has a value of $0.10. Adding one tricycle and one bicycle equals $0.50. Adding 2 bicycles and 1unicycle also equal $0.50.